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The Development of Spatial Skills through Discovering in the Geometrical Education at Primary School

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Abstract

There is a strong connection between 3-D and 2-D visualization of the geometrical shapes and solids. For better understanding of 3-D solids properties, it is important to have enough opportunities for manipulation with their 2-D representations, such are solid nets. According to the van Hiele´s theory, the development of geometrical thinking depends on the educational process, especially at school. We focused on the discovery activities with the 3rd and 7th grade pupils at primary school. We chose the cube and its nets as the most familiar 3-D solid in this age. We preferred IBL pedagogies and manipulation with the Polydron as the geometric construction product. We present some analysis of pupils´ work and their results when they were finding, sorting and repairing the cube nets. Through presented activities, spatial skills and geometrical conceptions of pupils are developed.

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Keywords: Spatial skills; Geometrical thinking; Cube nets; Inquiry based learning

1. Theoretical background

Spatial perception accompanies man from birth. Its development is connected with the cognitive processes but also with education. One of the possible and appropriate activities are manipulations with geometrical 2-D and 3-D shapes. As Linn and Petersen (1985) indicated that spatial ability is not a unitary construct, but it is combination of sub-skills such as using maps, solving geometry questions, and recognizing two dimensional representation of three-dimensional objects. Development of spatial cognition which entails the ability to mentally represent spatial relations and to anticipate the course and outcome of transformations applied to those relations has long attracted the

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interest of behavioral scientists (Rosser, 1995). Piaget and Inhelder (1967) defined two types of spatial ability when a child interacts with his/her environment. Perceptual Spatial Ability, the ability to perceive the spatial relationships between objects; and Conceptual Spatial Ability, the ability to build and manipulate a mental model of the environment. According to those researchers, children progress through three stages in the development of their cognitive spatial ability: preoperational stage, concrete operational stage, and formal operational stage (cited in Yilmaz, 2009).

Without geometric experiences, most people do not grow in their spatial sense or spatial reasoning. Between 1990 and 1992, NAEP data indicated a significant improvement in students' geometric reasoning at all three grades tested, 4, 8, and 12 (Strutchens & Blume, 1997). Students did not just get smarter. What is more likely is that there has been an increasing emphasis on geometry at all grades. In the study of Hassan (2002) was found that there was significant relationship between visual perception of geometric shapes and achievement of secondary school students in geometry. The results of study of Guzel and Sener (2009) show that spatial ability (three-dimensional thinking) improves students' understanding of symbols, shapes, tables, and figures. Besides, it assists students in comprehending drawings easily, commenting the visualized information, creating contexts among different concepts easily, generalizing complex concepts, and thinking in different ways. Accordingly, spatial ability plays crucial role to be successful in mathematics, specifically in geometry, for the reason that the field is based on visualization.

According to the theory of van Hiele (1999), there are five levels of understanding spatial concepts through which children move sequentially on their way to geometric thinking: visualization, analysis, abstraction, deduction and rigor. Van de Walle (2001) mentioned four characteristics of these levels of thought:

- The Van Hiele´s levels of geometric reasoning are sequential. Students must pass through all prior levels to arrive at any specific level.
- These levels are not age-dependent in the way Piaget described development.
- Geometric experiences have the greatest influence on advancement through the levels.
- Instruction and language at a level higher than the level of the student may inhibit learning.

The van Hiele theory indicates that effective learning takes place when students actively experience the objects of study in appropriate contexts, and when they engage in discussion and reflection. According to the theory, using lecture and memorization as the main methods of instruction will not lead to effective learning. Teachers should provide their students with appropriate experiences and the opportunities to discuss them (Mason, 1999).

It follows, that the school achievements of pupils in mathematics depend also on the teaching – learning approaches and methods. According to Rumanova et al (2014), pupils must build not only their knowledge, but also an active approach to learning itself.

One of the most familiar approach for pupils connected with discovering, discussing, questioning, handling in geometry is Inquiry-based learning (IBL). IBL is based on the constructivist and socio-constructivist theories of learning. Interest about IBL in the Europe increased during last ten years, mainly due to the strategic document Science Education NOW: A Renewed Pedagogy for the Future of Europe (Rocard, 2007). The European Union responded to a critical drop in the interest of young people about mathematics and science in this document. As a solution proposes the inclusion of new methods of teaching in schools, namely inclusion revelatory teaching approaches to the teaching mathematics and science subjects (Sikko et al, 2012). Inquiry-based learning involves exploring the world, asking questions, making discoveries, and rigorously testing those discoveries in search of new understanding. Inquiry-based learning can have many faces, dependent on context, target group and learning aims. However, inquiry-based learning approaches all have the shared characteristics of aiming to promote curiosity, engagement and in-depth learning. Inquiry-based learning aims to develop and foster inquiring minds and attitudes that are vital for pupils and learning is based on pupils adopting an active, questioning approach. (Primas project).

There are a lot of common features and approaches between IBL and van Hiele´s phases of learning. According to the van Hieles, a student progresses through each level of thought as a result of instruction that is organized into five phases of learning:

- Information: Through discussion, the teacher identifies what students already know about a topic and the students become oriented to the new topic.
- Guided orientation: Students explore the objects of instruction in carefully structured tasks such as folding, measuring, or constructing. The teacher ensures that students explore specific concepts.
- Explicitation: Students describe what they have learned about the topic in their own words. The teacher introduces relevant mathematical terms.
- Free Orientation: Students apply the relationships they are learning to solve problems and investigate more open-ended tasks.
- Integration: Students summarize and integrate what they have learned, developing a new network of objects and relations. (Mason, 1999)

2. Investigation of pupils´ geometrical skills

Theoretical bases mentioned above were implemented to the investigation of pupil's ability to find all cube nets and to solve related problems. The geometric construction product Polydron was used for manipulation with polygons consisted from six identical squares. We worked with two groups of pupils. The task for 3rd graders was to find all different polygons from which is possible to create a cube. The motivation was designing the dress for Ms. Cube from various patterns. The task for 7th graders was to find all cube nets and to solve a set of tasks connected with the cube and its nets. The aim of this part of the article is to show some possible and appropriate approaches, activities and tasks that should be implemented into school education of geometry in accordance with theoretical background. Through discovering processes in presented activities and tasks, pupils' spatial skills and geometrical knowledge were improved.

The sample of pupils' characterization

According to van Hiele, the eight years old pupils in the 3rd class at the elementary school, should be on the second level of geometrical thinking (analysis). In this level pupils see figures as collections of properties; can recognize and name properties of geometric figures, but they do not see relationships between these properties; when describing an object, a pupils might list all the properties they knows, but not discern which properties are necessary and which are sufficient to describe the object (cited in Mason, 1999). According to Piaget and Inhelder (1967), the second stage is the concrete operational stage, which occurs when children are between seven to nine years old. In this stage they develop a cognitive map with a fixed frame of reference, which allows them to imagine a view and orientation outside their body. Children develop an understanding of more complex topological relations using an external frame of reference, such as order and enclosure, and they begin to develop projective relations, like before/behind, and left/right. There is no topic about cube nets in the curriculum of geometry stated in The National Educational Program (NEP) in Slovakia. But presented activity in which the pupils were finding and drawing a cube nets with the support of handling and using a kit, showed to be appropriate for the children in this age.

The pupils in the 7th class at the elementary school should be on the third level of geometrical thinking (abstraction) according to van Hiele. In this level pupils perceive relationships between properties and between figures; can create meaningful definitions and give informal arguments to justify their reasoning; logical implications and class inclusions, such as squares being a type of rectangle, are understood; the role and significance of formal deduction, however, is not understood (cited in Mason, 1999). According to Piaget and Inhelder (1967), the third stage of cognitive development in childhood is the formal operational stage, which begins around the age of 11. In this stage, children develop a coordinate frame of reference, where individual routes blend into a network of locations in fixed positions relative to each other. They develop an understanding of Euclidean spatial relations, such as estimating straight-line relative distances, and proportional reduction of scale. According to The National Educational Program (NEP) in Slovakia, there is greater emphasis on the development of geometrical skills in the space in the 7th grade at the elementary school. Among the mathematical skills that should be gained by pupils in this age, is ability to scribe the cube and the prism in the parallel projection and to draw their nets.

Description and the evaluation of the activities

Activity with the 3rd graders

As was mentioned, pupils were finding all cube nets by manipulation with the six squares from the kit Polydron. In this age pupils do not need to know the terms: cube net and identity. But these concepts could be explained by the teacher after their discovering by the pupils during activity and be included to the pupils´ geometrical knowledge. At the beginning of the activity, pupils were discussing about the cube and its characteristics, teacher asked pupils whether they could create the cube from the building kit. Pupils were given to a role of the fashion designers and were asked to design various patterns for the Cube clothes. They worked themselves and everyone drew all discovered patterns on the paper. Later all patterns were compared with each other. Pupils were asked to decide, if all their patterns are really different or there are some the same after rotation or some other shifting. Next day we continued during the lesson of art education with designing the picture (mind map) from various cube nets cut from the paper. Pupils worked in groups and could express their creativity on the geometrical background. Teacher was only facilitator who guides the pupils´ discovering process and answers to pupils´ questions not directly, but through more guiding questions.

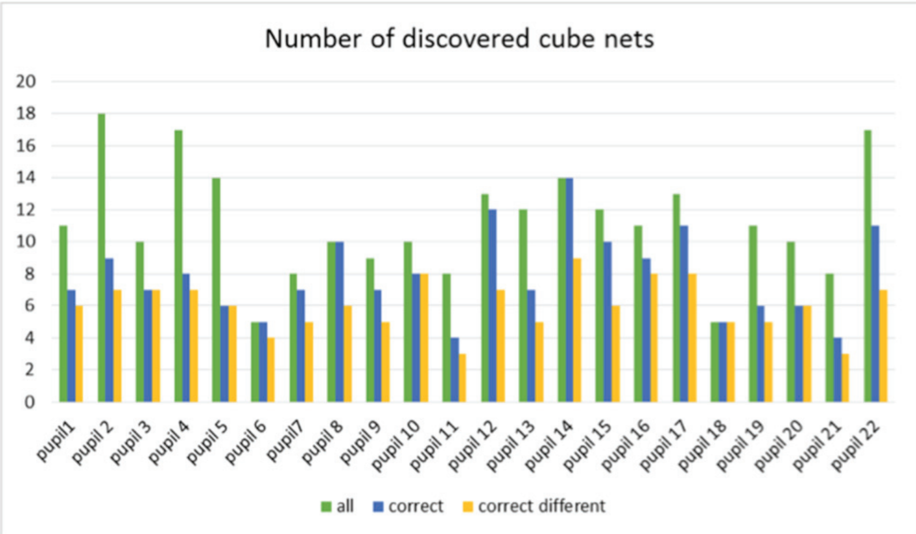

Fig. 1. Number of discovered cube nets

We were investigated the ability of pupils to find the most cube nets and to sort them according their identity. We evaluated all discovered cube nets by each pupil, correct and incorrect cube nets and different correct cube nets. There were 22 pupils in the class. Number of pupil's solutions is represented in the graph (Fig. 1.). As we can see, pupils were able to find enough cube nets, but not every were correct. It was interesting to search identical nets which were possible to cover each other after changing their position. It is important for future development of the concepts about similarity and identity. Handling with the Polydron helped pupils in their searching and a big role in this activity played motivation too. Pupils were solving geometric problem within working as a designer through discovering process. The activity continued next day with creating pictorial mind maps (Fig. 2.). According to Wang Ch-W. et al (2010), it is known that mind mapping increases creativity of young children and allows cognition of children can be easily understood. Mind map is a presentation form of radiant thinking, utilizing lines, colors, characters, numbers, symbols, images, pictures or keywords, etc. to associate and integrate, visualize the learned concept and maximize brain potential (Buzan & Buzan, 1996). Pupils created posters showing ZOO, heart, submarine, ship, nature, mosaic. Presented activity showed to be motivating and appropriate for 3rd graders though was new for them.

Fig. 2. Pictorial mind maps of 3rd graders

Activity with the 7th graders

According to the NEP, Slovak pupils in the 7th grade of elementary school should know only two from eleven cube nets. So, discovering another different nets is interesting for them and can help to increase their spatial skills. In this age is appropriate to solve various tasks on the paper by using mental manipulation with the cube or other 3-D shapes. Solving the problems written on the paper expects greater demands on pupil's mental processes and spatial skills. The pupils in our activity should solve six tasks with the cube nets background and could use the kit Polydron if they need. They could find solutions by handling or use their own mental spatial skills, could draw and compare their drawings with figures from kit. The tasks were focused on: 1. the identification and completion of the cube nets; 2. the identification of the opposite sides of a cube and their finding in the cube nets; 3. the identification of the edges of a cube and their finding in the nets; 4. the assignment of nets to a cube according to the different colored sides and the inverse task; 5. the completion of composing a cube from given nets; 6. the assignment of a cube to its nets according to line drawn across the sides and edges of a cube. We were also interested in pupils' opinions about difficulty of the tasks.

We were investigated the correctness of the pupils' solutions of the tasks, necessary of pupils handling with the Polydron and the mistakes and problems in the each task solution. The results of evaluation of pupils' solutions are in the Fig. 3. We have stated the overage percentage in each task. As we can see, the best pupils´ solutions were in the Task 1, in which they had to draw four different cube nets and to repair incorrect cube nets. But there were also solutions with the identical cube nets as it was in the 3rd graders results. We can say, that some pupils still did not have fixed concept about identical shapes.

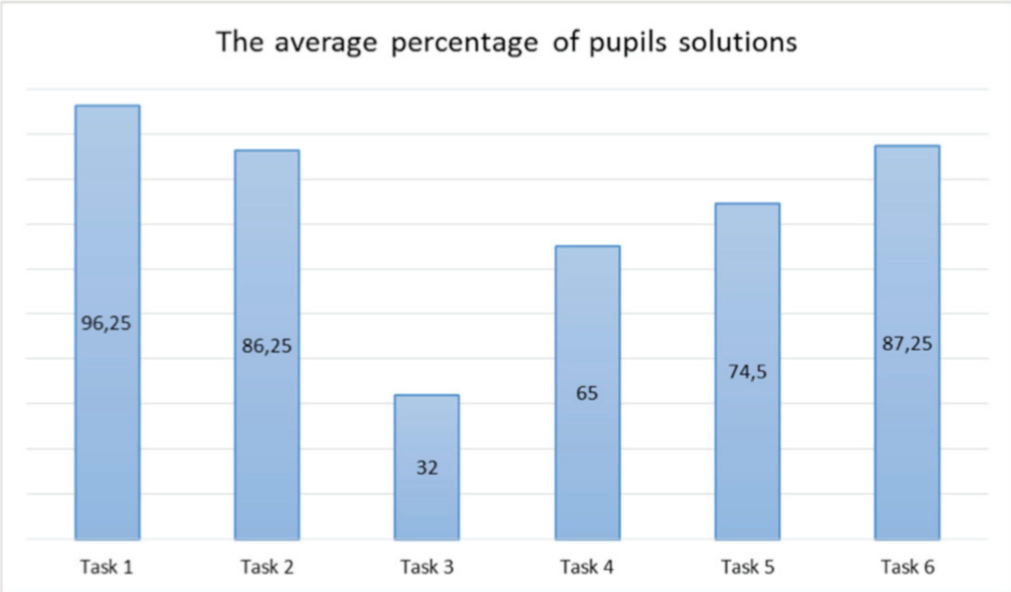

Fig. 3. The evaluation of pupils´ solutions

The most demanding task was Task 3. In this task pupils were asked to mark the same numbers to the pair of the sides in the cube net which join in one edge of a cube. One of the solution is in the Fig. 4.

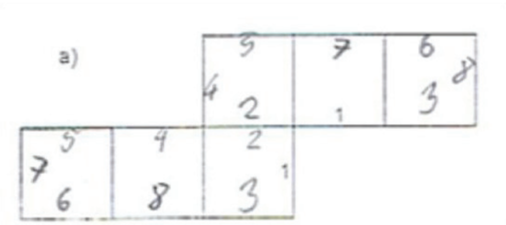

Fig. 4. Pupil´s solution of the Task 3

Pupils often manipulated with the Polydron when solving this task. The biggest problem was with understanding of the task. Misconceptions about sides of a square and edges of a cube appeared here. The most familiar task was Task 6 in which pupils had to choose correct net to the cube according to polyline drawing on the cube sides and edges (Fig.5.)

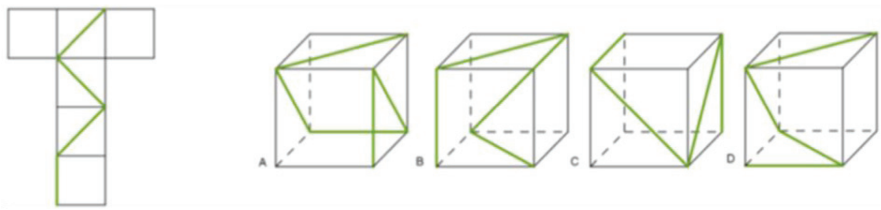

Fig. 5. Task 6

According to the analysis of pupils work and solutions of the tasks we can conclude that total percentage of the test was 73. 54% that can be considered satisfactory for us. Pupils were more familiar with the task focused on the working with the composing and decomposing of the cube nets. From the mental rotation point of view, for pupils was easier to manipulate with the sides of a cube more that with the edges or vertex. It can be concluded that our tasks were appropriate for 7th graders instead the fact, that they did not have any bigger experiences with presented type of activity and tasks. The lack of handling skills appeared but manipulation with Polydron helped pupils to find their own ways how to solve given problems. The pupils were satisfied at the end of their work and expressed positive feedback to the solved tasks.

3. Conclusions

There are two main conceptions how to teach geometry from early school years. First one is stated on the Euclidian theory and education starts with the basic concepts: point, line, plane, half-line, segment and continues with space shapes and solids. This approach not follows children's preschool experiences connected with playing, manipulation, handling with a cube, solids, box of bricks etc. The interruption of this intuitive way which influence children's skills to explore and examine 3D solids and their properties can have negative impact on the future development of their geometrical knowledge and skills. Second one is stated on the constructivist approach, follows children's experiences with 3D solids and continues with building new concepts about point, line, segment etc. based on the usage of space models. The first mentioned conception forms the basis of school geometric education in Slovakia. But there is effort to improve teaching methods and to support discovering and handling in geometric education in recent years.

The aim of the paper was to point out importance of the teaching – learning methods that influence the possibility of pupils to gain informal knowledge. Two similar activities with the same conceptual background with emphasis on the cognitive and geometric level of thinking of pupils were showed. Our investigations also confirm the fact that pupils of certain age, assuming the appropriate level of spatial skills and geometrical thinking, build and develop skills to solve geometrical problems by choosing their own solution strategies. It is very important for pupils to gain informal knowledge which will form the strong foundation for their further study.

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